SUBCOOLED FORCED-CONVECTION FILM BOILING IN THE FORWARD STAGNATION REGION OF A SPHERE OR CYLINDER

MICHAEL EPSTEIN and GEORGE M. HAUSER

Reactor Analysis and Safety Division, Argonne National Laboratory,
Argonne, IL 60439, U.S.A.

Abstract – An analysis is made of forced-convection film boiling in stagnation flow of subcooled liquids. The role of liquid viscosity in film boiling is determined by postulating the existence of a hydrodynamic boundary layer superposed on potential flow and using a perturbation technique. The viscous boundary layer due to shear stress at the vapor–liquid interface is shown to perturb the velocity field only slightly at large liquid subcooling. While the inviscid solution cannot be used to describe liquid motion when the liquid temperature is near its saturation temperature, the vapor is found to move only under the influence of the potential flow pressure distribution, thereby eliminating the coupling between the liquid boundary layer and vapor film without any significant errors in the heat-transfer problem. A rational interpolation formula between these two limiting cases leads to a simple expression for the film boiling heat transfer incorporating the major effects of wall superheat and liquid subcooling. The applicability of this formula to subcooled film boiling from a sphere or a cylinder is demonstrated.

NOMENCLATURE

a_l, stagnation point velocity gradient for the liquid [$3V_\infty/(2R)$ for a sphere];
A, dimensionless wall superheat parameter [equation (17)];
a, stagnation point velocity gradient for the vapor film [equation (9)];
B, dimensionless liquid subcooling parameter [equation (17)];
c, heat capacity;
C, integration constant for velocity solution in vapor film;
C_l, integration constant for velocity solution in liquid;
f, dimensionless vapor film stream function [equation (7)];
F, dimensionless liquid flow stream function [equation (6)];
h, heat-transfer coefficient;
k, thermal conductivity;
L, latent heat of liquid evaporation;
Nu, Nusselt number [$2Rh/k$];
p, pressure in the vapor film;
P, pressure in liquid;
P_∞, pressure in liquid far from sphere or cylinder;
Pr, Prandtl number;
R, radius of sphere or cylinder;
Re, liquid flow Reynolds number [$2RV_\infty/v_l$];
t, temperature in vapor film;
T, temperature in liquid; T_w, wall temperature; T_∞, free-stream temperature; T_{sat}, boiling temperature of the liquid;
u, U, vapor and liquid velocities in the x-direction, respectively;

v, V, vapor and liquid velocities in the y-direction, respectively;
V_x, free-stream velocity;
x, arc length measured from the front stagnation point [Fig. 1];
y, coordinate normal to the surface [Fig. 1].

Greek symbols

β, $(v\rho^{1/2})^{1/2}$ ratio [equation (17)];
δ, vapor film thickness [Fig. 1];
ε, density ratio [$(\rho/\rho_l)^{1/2}$];
η, dimensionless similarity coordinate for vapor film [equation (7)];
η_δ, dimensionless vapor film thickness [$\delta(2a/v)^{1/2}$];
$\eta_\delta(\theta)$, dimensionless vapor film thickness outside the stagnation region;
θ, dimensionless vapor film temperature [equation (10)];
μ, absolute viscosity;
v, kinematic viscosity;
ξ, dimensionless similarity coordinate for the liquid [equation (6)];
ρ, density;
ϕ, dimensionless liquid temperature [equation (10)].

Subscripts

l, liquid properties.

1. INTRODUCTION

THE FIRST theoretical study of upward-flow forced-convection film boiling from a horizontal cylinder was that of Bromley, LeRoy and Robbers [1]. They

examined the saturated liquid case and modified natural convection film boiling theory [2] to account for the effects of liquid velocity on the pressure distribution imposed on the vapor film. In developing their model, Bromley et al. assumed a constant evaporation rate per unit area of cylinder surface to evaluate the viscous drag at the vapor–liquid interface and that the interfacial velocity can be determined from potential flow theory. Because the results of their approximate solution did not correctly predict the functional behavior of their experimental data, they proposed an empirical relationship for predicting the rate of heat transfer. Motte [3] and Motte and Bromley [4] experimentally investigated forced-convection film boiling of subcooled liquids outside single cylinders over a wide range of temperatures and liquid velocities. These workers concluded that, while heat is transported across the vapor film by conduction, heat is transferred into the subcooled liquid by eddy motion (turbulence). They applied this idea to correlate their data; however, their correlation is dependent on their flow system and lacks generality.

Kobayasi [5] presented an analysis of film boiling on a sphere in a flowing saturated liquid in a manner identical to that of Bromley et al. [1] for a cylinder. In an attempt to simplify Kobayasi's analysis, Witte [6] ignored the pressure field established in the vapor film by the flowing liquid and, therefore, assumed the velocity profile within the vapor film is linear. Such an assumption can not be valid since, owing to the low density of the vapor compared to that of the liquid, pressure gradients must very significantly influence the velocity profile in the vapor film. Recently, Dhir and Purohit [7] performed experiments on heat transfer during subcooled film boiling of water on spheres. They cited some values of the heat-transfer coefficient in the forced convection regime.

Additional experimental work, in which high-temperature metal spheres were passed rapidly through water, has been reported by Walford [8] and Stevens and Witte [9]. These experiments were designed to study the transition from film to nucleate boiling. Walford [8] measured an average heat flux which apparently includes contributions from both film and nucleate boiling, while in the experiments by Stevens and Witte [9] the liquid was significantly subcooled so that either stable film boiling did not occur or quickly gave way to nucleate boiling. Work with a hot sphere quenched in liquid sodium is described in [10]. The liquid subcooling in these experiments was so extreme that it is doubtful that film boiling took place at all.

An objective of this paper is to present a logical theoretical treatment of film boiling of flowing liquids on spheres or cylinders with due consideration of the effects of liquid subcooling and viscosity. This analysis approaches the problem from the point of view that the vapor film thickness is a weak function of the arc length measured from the front stagnation point. The nature of the heat transfer from the sphere or cylinder

may thus be determined from the solution of the governing equations for stagnation flow. If one has information regarding the location at which the vapor film is transformed into a thick vapor wake (separation point), then one can easily solve for the heat-transfer coefficient. This information may be obtained from experimental observation or inferred from consideration of the potential flow pressure gradient.

Strictly speaking, the vapor film thickness is not constant over the surface of the sphere but grows over the leading portion of the body and may rapidly increase by over a factor of two near the equator of the sphere. Thus, we anticipate that a film boiling model based on a constant vapor film thickness will overestimate the heat-transfer rate. However, as will be seen, the vapor film thickness variation over most of the front surface of the sphere is small enough so that the heat-transfer coefficient is not appreciably in error by assuming a uniform vapor film. One can readily develop a model that includes the effects of vapor film growth with distance from the stagnation point (see Section 4.2). Unfortunately, while conceptually simple, such models require numerical calculations and lack generality. The present treatment, based on the notion of a uniform vapor film, suggests a universal correlating expression for forced-convection film boiling heat transfer that incorporates the effects of both wall superheat and liquid subcooling.

2. FILM BOILING IN AXIALLY SYMMETRICAL STAGNATION FLOW

2.1. Physical model

Let us consider a stationary solid sphere of radius R situated in an upward-flowing, unbounded liquid which has a uniform, constant velocity V_∞ at infinity.
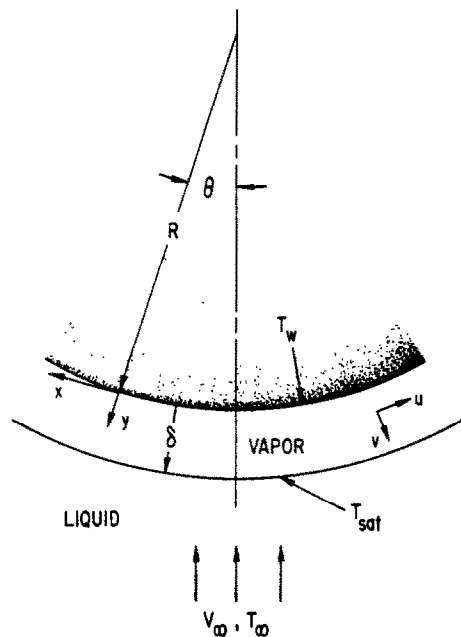

FIG. 1. Physical model and coordinate system for film boiling in the forward stagnation region.

We shall restrict ourselves to the forward stagnation region of the sphere as illustrated in Fig. 1. The temperature of the liquid at infinity is T_∞ which is lower than the liquid saturation temperature T_{sat}. The temperature of the sphere is sufficiently high so that the sphere is separated from the liquid by a continuous vapor film of uniform thickness δ. We introduce a curvilinear system of coordinates as shown in Fig. 1. We denote the arc length measured along any meridian in the θ-direction from the front stagnation point (located at the bottom of the sphere) by x, and y is the coordinate normal to the surface of the sphere, outward as positive.

The model of the vapor film used in the present analysis is essentially the one presented by Bromley et al. [1]. Accordingly, in addition to the assumption of a vapor film of uniform thickness, the following assumptions are made:

(A1) The inertia terms in the vapor film momentum equation and the convection terms in the vapor film energy equation can be neglected.

(A2) The vapor film is very thin compared with the radius of the sphere.

(A3) The vapor flow is laminar around the sphere and the vapor–liquid interface is smooth.

(A4) The surface temperature of the sphere, T_w, is taken to be uniform.

(A5) All physical properties of the liquid and its vapor are assumed to be constant and may be evaluated at the arithmetic average temperature $(T_w + T_{sat})/2$ for the vapor and $(T_{sat} + T_\infty)/2$ for the liquid.

To extend the model of [1] to include liquid subcooling, it is necessary to consider energy convection within the liquid. To accomplish this, we make the following additional assumptions:

(A6) The region in which the temperature of the liquid varies appreciably is a thin one adjacent to the vapor–liquid interface; in other words, the thermal boundary layer in the liquid is small compared to the radius of the sphere.

(A7) The liquid velocity is high enough for the gravitational force (buoyancy) to be negligible.

(A8) Radiation heat transfer is neglected.

A discussion of the expected accuracy of these approximations is postponed to Section 4.2.

2.2. Basic conservation equations

The underlying conservation equations incorporating the foregoing assumptions are those for laminar,

axisymmetric stagnation flow. Across the vapor–liquid interface at $y = \delta$, velocity, shear and mass flow continuity requirements provide the relations

$$u = U, \quad \mu \frac{\partial u}{\partial y} = \mu_l \frac{\partial U}{\partial y}, \quad \rho v = \rho_l V. \tag{1}$$

In addition, temperature and energy continuity is also required at this interface

$$t = T = T_{sat}, \quad k \frac{\partial t}{\partial y} = \rho v L + k_l \frac{\partial T}{\partial y}. \tag{2}$$

Capital letters are used to identify the liquid velocity and temperature variables, while those in the vapor film are assigned lower case letters. Unsubscripted physical properties pertain to the vapor film, while those in the liquid region bear the subscript l. The boundary conditions imposed at the hot surface of the sphere and in the liquid flow at infinity are

$$y = 0, \quad u = v = 0, \quad t = T_w; \tag{3}$$

$$y \to \infty, \quad U \to a_l x, \quad V \to V_\infty = -2a_l y, \quad T \to T_\infty; \tag{4}$$

$$P \to P_\infty + \frac{1}{2} a_l^2 \rho_l \left(\frac{4}{9} R^2 - x^2 \right). \tag{5}$$

Equations (4) and (5) simply express the fact that the liquid flow field and pressure distribution away from the thin viscous boundary layer is taken to be that for potential flow. The constant a_l used above is the stagnation point velocity gradient. For flow in the front stagnation point region of a sphere $a_l = 3V_\infty/(2R)$.*

Now, as usual in boundary layer theory, the quantities in the above equations are non-dimensionalized. In particular, for the liquid region momentum field, we define the variables

$$\xi = (2a_l/v_l)^{1/2} y, \quad U = a_l x \frac{dF(\xi)}{d\xi},$$

$$V = -(2a_l v_l)^{1/2} F(\xi). \tag{6}$$

Similarly, for the vapor film there is introduced

$$\eta = (2a/v)^{1/2} y, \quad u = ax \frac{df(\eta)}{d\eta},$$

$$v = -(2av)^{1/2} f(\eta), \tag{7}$$

where the pressure distribution in the vapor film is given by

$$p = P_\infty + \frac{1}{2} a^2 \rho \left(\frac{4}{9} R^2 - x^2 \right). \tag{8}$$

Since the pressure distribution in the vapor film is precisely the same as that in potential flow – a consequence of the nature of boundary layers, the stagnation point velocity gradient 'a' for the vapor film is related to a_l according to

$$a = (\rho_l/\rho)^{1/2} a_l \tag{9}$$

[compare equation (5) with equation (8)].

---

* The present analysis can be applied to predict film boiling heat transfer from a hot surface in the impingement region of a liquid jet by simply setting $a_l \simeq V_\infty/d$ where d is the jet diameter. In the Jominy end-quench test for the determination of the hardenability of steel, a round steel bar is heated above 910°C and is then end-quenched with a water jet (see, for example, [11]). Early in the quenching period, the jet is in film boiling.

Finally, the temperature profiles are conveniently described in terms of the following dimensionless choice:

$$\phi = \frac{T(\zeta) - T_{sat}}{T_x - T_{sat}}, \quad \theta = \frac{t(\eta) - T_w}{T_{sat} - T_w}. \quad (10)$$

The liquid momentum and energy conservation equations now reduce to the well known forms

$$F''' + FF'' + \frac{1}{2}(1 - F'^2) = 0, \quad (11)$$

$$\phi'' + Pr_l F \phi' = 0, \quad (12)$$

where the primes denote differentiation with respect to the independent variable ζ. For the vapor film there results

$$f''' + ff'' + \frac{1}{2}(1 - f'^2) = 0, \quad (13)$$

$$\theta'' + Prf\theta' = 0 \quad (14)$$

with the primes representing differentiation with respect to η.

With this choice of notation, the matching conditions at the vapor–liquid interface transform to

$$F(0) = \varepsilon\beta f(\eta_\delta), \quad F'(0) = f'(\eta_\delta)/\varepsilon,$$

$$F''(0) = \beta f''(\eta_\delta), \quad (15)$$

$$\phi(0) = 0, \quad \theta(\eta_\delta) = 1.0,$$

$$A\theta'(\eta_\delta) = f(\eta_\delta) + B/(Pr_l)^{1/2}\phi'(0), \quad (16)$$

where ε, β, A and B are dimensionless parameters defined as

$$\varepsilon \equiv \left(\frac{\rho}{\rho_l}\right)^{1/2}, \quad \beta \equiv \left[\left(\frac{\rho}{\rho_l}\right)^{1/2}\frac{\nu}{\nu_l}\right]^{1/2}, \quad A \equiv \frac{c(T_w - T_{sat})}{PrL},$$

$$B \equiv \beta\frac{k_l}{k}\frac{c(T_{sat} - T_\infty)}{PrL}(Pr_l)^{1/2}. \quad (17)$$

Note that in equations (15) and (16) the value of η at the vapor–liquid interface is denoted by $\eta_\delta = \delta(2a/\nu)^{1/2}$. For convenience, we take $\zeta = 0$ at the vapor–liquid interface. This is permissible because ζ does not enter into the matching conditions. The parameter A in definition (17) is the familiar dimensionless wall superheat, while B is a parameter related to the degree of liquid subcooling.

Applying the transformations (6), (7) and (10) to the boundary conditions (3) and (4) yields

$$\eta = 0: \quad f(0) = f'(0) = 0, \quad \theta(0) = 0, \quad (18)$$

$$\zeta \to \infty: \quad F'(\zeta) \to 1.0, \quad \phi(\zeta) \to 1.0. \quad (19)$$

The ordinary differential equations (11)–(14), together with the matching and boundary conditions (15), (16), (18) and (19) suffice to determine the four unknowns. The problem is seen to involve consider-

able mathematical complexity and a large number of independent physical parameters. However, a significant simplification in the problem can be obtained by the approach to be followed here.

2.3. Solution method and results for velocity field

According to assumption (A1), we can set aside the effects of inertia and energy convection in the vapor film. With this, equations (13) and (14) simplify to

$$f''' = -\frac{1}{2}, \quad \theta'' = 0. \quad (20)$$

Solutions of these equations satisfying the boundary conditions (18) and the second of (16) are

$$f = -\frac{1}{12}\eta^3 + \frac{1}{2}C\eta^2, \quad \theta = \frac{\eta}{\eta_\delta}, \quad (21)$$

where the integration constant C and the film thickness η_δ remain to be determined from the interface matching conditions and the liquid velocity and temperature fields.

Compare the flow of liquid in film boiling over a hot sphere with that of a sphere in thermal equilibrium with the surrounding liquid. For the latter case, the boundary layer is perceived as a thin layer at the sphere surface where viscous forces play a dominant role; outside this layer, the flow departs little from the inviscid flow pattern. For the hot sphere covered with a vapor film, the liquid does not come to rest anywhere within the flow field. At first sight it might appear that potential flow could be a valid solution for the entire liquid flow field [1, 5, 6]. This is not always the case, however, as we shall point out below. Moreover, the velocity derivatives for potential flow would not satisfy the tangential stress boundary condition at the vapor–liquid interface. Assuming the liquid flow is only slightly perturbed from potential, a first approximation for the velocity in the liquid boundary layer is

$$F(\zeta) = \zeta + W(\zeta). \quad (22)$$

The perturbation function $W(\zeta)$ is assumed to be small such that $|W(\zeta)| \ll \zeta$. Substituting this expression into equation (11) and deleting terms of order W^2 gives the following linearized momentum equation:

$$W''' + \zeta W'' - W' = 0 \quad (23)$$

subject to the matching conditions [see equation (15)]

$$W(0) = \varepsilon\beta f(\eta_\delta), \quad 1.0 + W'(0) = f'(\eta_\delta)/\varepsilon,$$

$$W''(0) = \beta f''(\eta_\delta) \quad (24)$$

and the boundary condition [see equation (19)]

$$W'(\infty) \to 0. \quad (25)$$

The solution of equation (23) which satisfies condition (25) at infinity is*

$$W' = C_l\left[e^{-\zeta^2/2} - \sqrt{\frac{\pi}{2}}\zeta\,\mathrm{erfc}\,(\zeta/2^{1/2})\right], \quad (26)$$

where C_l is an integration constant. Inasmuch as the

---

* This solution is obtained by using the transformation $W'(\zeta) = \zeta S(\zeta)$, which results in the separable equation $S'' + (2\zeta^{-1} + \zeta)S' = 0$.

present analysis is developed under the assumption that the perturbed liquid velocity is small as compared to the velocity given by the inviscid solution, equation (26) will not be valid unless C_l is a small quantity.

Utilizing the second and third matching conditions (24) and equations (21) and (26) then gives the following algebraic relations between the constants of integration and the vapor film thickness, η_δ:

$$C = \frac{\frac{1}{4}\eta_\delta^2 + \frac{\varepsilon\beta}{\sqrt{2\pi}}\eta_\delta + \varepsilon}{\eta_\delta + \sqrt{\frac{2}{\pi}}\varepsilon\beta}, \qquad (27)$$

$$C_l = \frac{-\frac{1}{4}\eta_\delta^2 - \varepsilon}{\varepsilon + \sqrt{\frac{\pi}{2}}\frac{1}{\beta}\eta_\delta}. \qquad (28)$$

The tangential component of the liquid velocity $F'(\xi)$ can readily be evaluated from equations (22) and (26)

$$F'(\xi) = 1.0 + \frac{\frac{1}{4}\eta_\delta^2 - \varepsilon}{\varepsilon + \sqrt{\frac{\pi}{2}}\frac{1}{\beta}\eta_\delta}$$

$$\times \left[e^{-\xi^2/2} - \sqrt{\frac{\pi}{2}}\,\xi\,\text{erfc}\,(\xi/\sqrt{2}) \right]. \quad (29)$$

Examination of this result reveals that $F'(\xi)$ becomes identical to the potential flow solution when $\eta_\delta = 2\varepsilon^{1/2}$. For thicker vapor films, $\eta_\delta > 2\varepsilon^{1/2}$, due to the free stream pressure distribution impressed on the vapor film, the vapor velocity exhibits a maximum within the film and exceeds the liquid velocity within some of the film. Frictional shear is transported from the vapor flow to the liquid flow. In this case the liquid velocity is higher than that given by the potential solution. Conversely, when the vapor film is thin (i.e. $\eta_\delta < 2\varepsilon^{1/2}$), then the tangential velocity in the liquid falls below that given by potential flow theory, indicating that frictional shear is transmitted from liquid to vapor. The liquid velocity is now larger than that in the vapor film; the form of the vapor velocity distribution is monotonic, decreasing from some finite value at the vapor–liquid interface to zero at the hot surface. In actual practice, the vapor film thickness never becomes much smaller than $2\varepsilon^{1/2}$. Thus, hereafter the term 'thin film' (or the thin-vapor-film limit) will refer to the rough equality $\eta_\delta \sim 2\varepsilon^{1/2}$.

The range of film thicknesses for which only small departures from the inviscid solution are to be expected can be calculated from the present results, with the help of some representative values of the parameters β and ε. The parameter β [see equation (17)] is generally a number of order unity. For instance, for water,

---

* Equation (32) is applicable to liquid metals by setting $C_l(\eta_\delta) = 0$ in the third term in equation (32).

ethanol, and sodium at one atmosphere, the values of β are 0.8, 1.5, and 2.6 respectively. On the other hand, the values of ε at one atmosphere are quite small and are in the range $0.01 < \varepsilon < 0.05$. If we require the perturbed velocity component to fall within, say, 20% of the potential flow value, equation (29) suggests that the vapor film thickness should be confined to the range $0.12 \gtrsim \eta_\delta \gtrsim 0.8$. Using the results of the next section, we find that in a highly subcooled liquid the vapor film thickness falls within this range. However, as subcooling is decreased and saturated conditions are approached, η_δ is predicted to exceed 0.8 by more than a factor of 3.0 resulting in appreciable departures from inviscid motion in the liquid. Fortunately, for liquid temperatures near saturation, we can neglect the existence of the viscous liquid boundary layer without causing any significant error in the heat-transfer problem. This will be demonstrated below.

2.4. Vapor film thickness

For high Prandtl number liquids, the thermal boundary layer will be restricted to a very thin region near the vapor–liquid interface, and we may, therefore, employ an expression for $F(\xi)$ which is applicable for small ξ. Expanding $F(\xi)$ in a Taylor series, it is found that

$$F(\xi) = F(0) + \xi F'(0) + \frac{1}{2}\xi^2 F''(0) + \ldots = \varepsilon\beta f(\eta_\delta)$$

$$+ \xi[1 + C_l(\eta_\delta)] - \frac{1}{2}\left(\frac{\pi}{2}\right)^{1/2}\xi^2 C_l(\eta_\delta) + \ldots, \quad (30)$$

where use has been made of the first of matching conditions (15). Since both C_l and ξ are small compared with unity, it is clear that we need only retain the first two terms in this series. Substituting this into equation (12) and integrating yields

$$\phi'(0) = \frac{[(2/\pi)Pr_l(1 + C_l)]^{1/2}}{e^b\,\text{erfc}\,(b^{1/2})}, \qquad (31)$$

where $b = (1/2)[\varepsilon\beta f(\eta_\delta)]^2 Pr_l/(1 + C_l)$. The values of b with which we have to deal are much less than unity. For example, for water in film boiling at one atmosphere $b \simeq 0.01$. Physically, this merely implies that the evaporation process has little effect on the liquid flow field and $F(0)$ can be taken to be zero.

Substituting equation (31), where $b = 0$ and equation (21) into the last of the matching conditions (16) gives the following implicit relationship for the vapor film thickness:

$$\frac{1}{6}\eta_\delta^4 - \frac{1}{2\beta}\left(\frac{\pi}{2}\right)^{1/2}\eta_\delta^3 C_l(\eta_\delta)$$

$$+ B\left(\frac{2}{\pi}\right)^{1/2}\left[1 + C_l(\eta_\delta)\right]^{1/2}\eta_\delta = A, \quad (32)$$

where the $C_l(\eta_\delta)$ relation is given by equation (28).* We are now in a position to derive an interpolation formula for the vapor film thickness which is valid for any liquid. Values of A in the range $0.1 < A < 5.0$ and

of B for $0 < B < 3.0$ have been obtained in the laboratory [1,3,4,7]. Predicted values of η_δ over this range are found to be remarkably insensitive to ε and β. As an example, we may note that if $\varepsilon = 0.03$ and $A = B = 1.0$ an increase in the magnitude of β from 0.1 to 10.0 results in a variation of less than 10% in the value of η_δ. This is to be compared with an increase in the interfacial velocity from $F'(0) = 1.0$ for $\beta = 0.1$ to $F'(0) = 2.0$ when $\beta = 10.0$ [see equation (29)]. Thus it appears that the film thickness perturbation is small even when the liquid velocity perturbation is not small which allows us to extend equation (32) into a parameter domain well beyond that for liquid motion. This behavior is readily explained by examination of equations (28) and (32). In the case of highly subcooled liquids, when $B \gg A$ both η_δ and $C_l(\eta_\delta)$ are small quantities. The third term in equation (32) dominates the first two terms and the film thickness takes the limiting form

$$\eta_\delta \rightarrow \left(\frac{\pi}{2}\right)^{1/2} \frac{A}{B} \qquad \text{for} \quad B \gg A, \tag{33}$$

which is independent of ε and β. In this limiting case, the liquid subcooling is so large that practically all of the energy transferred from the sphere heats the flowing liquid. The contribution of liquid evaporation to the energy balance at $\eta = \eta_\delta$ is negligible, and the vapor film thickness is dictated by the heat conducted through it. The reason for the validity of expression (33) for common liquids with large amounts of subcooling, is that the vapor film becomes sufficiently thin so that the liquid shear is small (i.e. $\eta_\delta \sim 2\varepsilon^{1/2}$) and the liquid heat transfer can be determined from potential flow theory. For nearly saturated liquids, $A \gg B$, η_δ is of order unity and $C_l(\eta_\delta)$ approaches the quantity $C_l = \beta\eta_\delta/(8\pi)^{1/2}$ [see equation (28)]. Substituting this value into equation (32) and neglecting the third term which is now small compared with the others, yields

$$\eta_\delta \rightarrow (24A)^{1/4} \qquad \text{for} \quad A \gg B, \tag{34}$$

an equation which is also independent of ε and β. This shows that under saturated boiling conditions the vapor flow is mainly governed by the potential flow pressure distribution outside the viscous layer, the effect of liquid motion being imperceptible. The vapor velocity within most of the film is so much larger than the liquid velocity that, as far as the vapor motion is concerned, the phase change boundary at $\eta = \eta_\delta$ acts as a solid wall. Recall that for thick vapor films the vapor velocity rises to a maximum within the film (at $\eta \sim \eta_\delta/2$ when $\eta_\delta \gg 2\varepsilon^{1/2}$).

By considering the structure of equations (33) and (34), it is not difficult to see that the following relation transforms into (33) and (34) when $A \gg B$ and $A \ll B$:

$$\eta_\delta = \left[\frac{1}{24A} + \left(\frac{2}{\pi}\right)^2 \left(\frac{B}{A}\right)^4\right]^{-1/4}. \tag{35}$$

Since η_δ increases linearly with A at high subcoolings and as the one-fourth power of A at low subcoolings,

the accuracy of this interpolation formula is relatively high. An implicit, but more accurate relationship between the vapor film thickness and the parameters A, B is obtainable from equation (32) by simultaneously setting $C_l = \beta\eta_\delta/(8\pi)^{1/2}$ in the second term and $C_l = 0$ in the third term, namely

$$\frac{1}{24}\eta_\delta^4 + \left(\frac{2}{\pi}\right)^{1/2} B\eta_\delta = A. \tag{36}$$

This relation could be obtained directly by assuming that the flow of vapor moves under the influence of the potential flow pressure field only and is contained between two stationary parallel planes (i.e. the liquid–vapor interface and the surface of the sphere), with the liquid heat transfer based on inviscid flow. In the region where both liquid subcooling and wall superheat are important, the vapor film thickness given by equation (35) is only about 15% different than that from the more accurate result (36).

Of course, equation (35) is only expected to be valid for physical parameters A and B leading to a stable vapor layer covering the hot surface. As an example, Dhir and Purohit [7] observed that the minimum surface temperature to sustain film boiling of water on a hot sphere does not depend on liquid velocity and can be correlated in K by $T_w - T_{sat} = 101 + 8(T_{sat} - T_x)$. In terms of our dimensionless parameters, this result leads us to expect stable film boiling for water when $A \gtrsim 0.08 + 0.3B$.

2.5. Heat transfer

Calculations based on the theoretical pressure distribution derived from potential flow theory indicate an adverse pressure gradient on the downstream side of bodies such as spheres and cylinders (i.e. for $\theta > \pi/2$). Thus, the vapor film cannot penetrate too far past the equatorial plane of the sphere without becoming very thick. At sufficiently high liquid flow velocities, visual observation indicates that this 'vapor film separation' occurs between positions $\theta = \pi/2$ and $3\pi/2$ radians from the forward stagnation point, depending on the degree of liquid subcooling [12]. The downstream side of the sphere is observed to be covered by a thick vapor wake. A similar observation is reported in [1] for film boiling from a horizontal cylinder with the liquid flow directed upward. Temperature measurements within the interior of the cylinder demonstrated that the largest percentage of heat is transferred on the lower half of the cylinder.

If the heat transfer above the angle $\theta = \pi/2$ is neglected, the heat-transfer coefficient, h, based on the total surface area of the sphere becomes

$$h = \frac{k}{2\delta} = \left(\frac{3}{2}\right)^{1/2} \cdot \frac{kRe^{1/2}}{2R\beta\eta_\delta}, \tag{37}$$

where $Re = 2RV_x/v_l$ is the liquid flow Reynolds number. Equation (37) is based on a uniform vapor film thickness over the lower surface of the sphere equal in value to that in the forward stagnation region.

The justification of doing this is presented below and in Section 4.2. The Nusselt number, $Nu = 2Rh/k$, may be expressed as

$$\frac{\beta Nu}{(Re)^{1/2}} = \left(\frac{3}{2}\right)^{1/2}\frac{1}{\eta_\delta} = \left(\frac{3}{2}\right)^{1/2}\left[\frac{1}{24A} + \left(\frac{2}{\pi}\right)^2\left(\frac{B}{A}\right)^4\right]^{1/4}. \tag{38}$$

As mentioned previously, solutions for film boiling of saturated liquids on spheres were provided by Kobayasi [5] and Witte [6]. Kobayasi's expression for high liquid velocity, when cast in the notation of Section (2.2), is given by

$$\frac{\beta Nu}{(Re)^{1/2}} = \frac{0.454}{A^{1/4}}. \tag{39}$$

The present method (of equation (38) when $B = 0$) yields the equation

$$\frac{\beta Nu}{(Re)^{1/2}} = \frac{0.553}{A^{1/4}}. \tag{40}$$

This relation has qualitative behavior which is identical to equation (39). The small difference in the numerical constants of 22% is due undoubtedly to our assumption of a uniform vapor film thickness. Witte's result, which ignores the impressed pressure distribution, takes the form

$$\frac{\beta Nu}{(Re)^{1/2}} = \frac{0.702\varepsilon^{1/2}}{A^{1/2}} \tag{41}$$

and yields values for Nu lower than equations (39) or (40) by about a factor of 4.0 for $A \simeq 1.0$. This result would not be expected to apply to most film boiling situations of practical interest.

3. FILM BOILING IN TWO-DIMENSIONAL STAGNATION FLOW

The analysis of film boiling in the stagnation region of a cylinder is similar to that for a sphere, the only basic difference being plane flow as opposed to axisymmetrical flow. Omitting the derivation, the approximate equation for the Nusselt number for forced convection film boiling from a cylinder incorporating the assumptions of Section 2.1 is

$$\frac{\beta Nu}{(Re)^{1/2}} = \left(\frac{3}{2}\right)^{1/2}\left[\frac{1}{27A} + \left(\frac{4}{3\pi}\right)^2\left(\frac{B}{A}\right)^4\right]^{1/4}. \tag{42}$$

The ratio of the Nusselt number for a sphere to that for a cylinder is between 1.03 and 1.22 depending on the amount of liquid subcooling. Considering the random scatter of the experimental data and the simplifying assumptions in the analysis, this small difference between the two geometries can be ignored.

4. DISCUSSION

4.1. Comparison with experiment

In Fig. 2, the present results are compared with available experimental data for forced-convection film boiling from cylinders and spheres [1, 2, 3, 4, 7]. Motte [3] and Motte and Bromley [4] studied film boiling with ethyl alcohol, hexane, carbon tetrachloride, and benzene from cylinders of 0.983, 1.26, and 1.62 cm diameter. The data represent subcoolings from 11.0–45°C and liquid flow velocities in the range 0.9 to 4.0 m s^{-1}. Data for film boiling of ethyl alcohol [1] at saturation temperature in the forced convection regime are also included in Fig. 2. Dhir and Purohit's [7] data were obtained for film boiling from a 1.9 cm diameter sphere in the boiling region very close to the minimum heat flux. The amount of water subcooling was varied from 0–50°C and the water flow velocity was increased from 0.02–0.45 m s^{-1}. To avoid the effects of gravity, Fig. 2 includes only the highest flow velocity reported in [7].

It is clear from Fig. 2 that the observed heat transfer rates are higher than the calculated values in every case. This is not unexpected since the model does not account for wave motion at the liquid–vapor interface, liquid–solid contact nor heat transfer above $\theta = \pi/2$. Nevertheless, the order of magnitude and the general trend of the data agree well with the theory. Comparison of the solution (38) with the experimental results presented in Fig. 2, shows that a numerical correction factor of about 2.0 should be introduced to best correlate the data (dashed curve in Fig. 2). It should be mentioned that a numerical correction factor of this magnitude has been introduced to
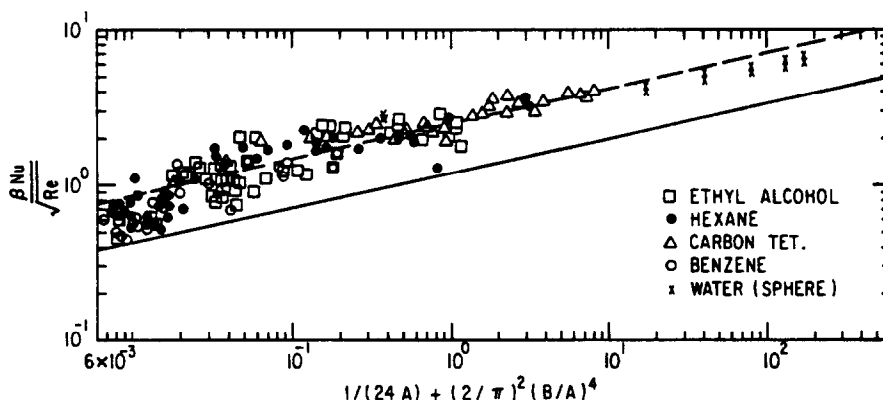

FIG. 2. Forced-convection film boiling from a horizontal cylinder and a sphere [1–4, 7]: ———, based on theory; ———, suggested correlation.

account for the intensifying effect of waves on heat transfer in film condensation of flowing vapor on a cold surface [13]. Note that the inverse of the vapor film thickness to the fourth power is used as the abscissa in Fig. 2 [see equation (35)]. Most of the measurements have been made in the thick vapor film boiling regime corresponding to low values of the abscissa. In this region, the vapor superheat parameter A dominates and liquid subcooling effects are negligible. The scatter in the data seems to suggest that the correlation improves with increased liquid subcooling, although this may very well be due to the paucity of data at high liquid subcooling. It would be desirable to obtain additional data in this region.

4.2. Validity of the model

First we focus attention on the applicability of equation (38) to film boiling on a sphere or cylinder. This information is provided by a more general analysis which allows for a variable vapor film thickness. An expression for the vapor film thickness as a function of the angle θ measured relative to the upstream axis of symmetry (see Fig. 1) can be obtained from a mass balance for a small differential element of vapor. Invoking the assumptions of Section 2.1, we find that $\eta_\delta(\theta)$ satisfies the ordinary differential equation

is given by equation (36) which is recovered by setting $d\eta_\delta(\theta)/d\theta = \theta = 0$ in equation (43). Equation (43) was solved numerically using a computer library program. Typical results for saturated and subcooled film boiling, showing the effects of wall superheat on the growth of the vapor film in the θ-direction, are displayed in Fig. 3. Note that the film thickness is normalized with respect to the minimum film thickness at the front stagnation point, η_δ. It can be seen that the normalized vapor film thickness is quite insensitive to A and B for sphere surface positions less than 1.0 rad. Note also that $\eta_\delta(\theta) \simeq \eta_\delta$ over most of the lower surface of the sphere. When the liquid subcooling effect is important (when $B > A$), the numerical integration could be carried out almost all the way to $\theta = \pi/2$ without a significant increase in the vapor film thickness (vapor film separation). This is illustrated in Fig. 3 for the cases $A = 0.2, A = 1.0$ when $B = 2.0$. Neglect of the variation of vapor film thickness with surface position results in overestimating the heat transfer by some 35% in the case of saturated liquids, but this error becomes less than 20% in the limit of large liquid subcooling.

The large predicted increase in the film thickness near the equator of the sphere is due to the lessening and reversal of the pressure gradient as θ approaches

$$\frac{d\eta_\delta(\theta)}{d\theta} = \frac{A - \frac{1}{24}\left(\cos^2\theta - \frac{1}{2}\sin^2\theta\right)\eta_\delta^4(\theta) - \left(\frac{3}{2\pi}\right)^{1/2} . B . (1 + \cos\theta)(2 + \cos\theta)^{-1/2} . \eta_\delta(\theta)}{\frac{1}{16}(\sin\theta\cos\theta)\eta_\delta^3(\theta)}$$

$$(43)$$

where, in accordance with what has been said earlier for stagnation flow, the vapor–liquid interface is assumed to be stationary as far as the vapor flow is concerned, and the vapor flow and heat transfer in the liquid are assumed controlled by the potential flow pressure distribution and velocity field, respectively. The initial condition for equation (43) comes from the observation that the vapor film curvature vanishes at the stagnation point $\theta = 0$. Thus, the initial value $\eta_\delta(\theta) = \eta_\delta$ at $\theta = 0$ required for the solution of the problem

and exceeds $\pi/2$. This is in qualitative agreement with available photographic sequences for film boiling of saturated liquids from spheres falling through water [12], which indicate that the vapor film thickness in the region $\theta \simeq \pi/2$ may be several times the vapor film thickness covering the front of the sphere. Near the equator of the sphere, within the angular ring $[(\pi/2) - \theta]$, the drag of the liquid on the vapor film overshadows the pressure gradient 'force' and the film motion is not unlike forced-convection film boiling of
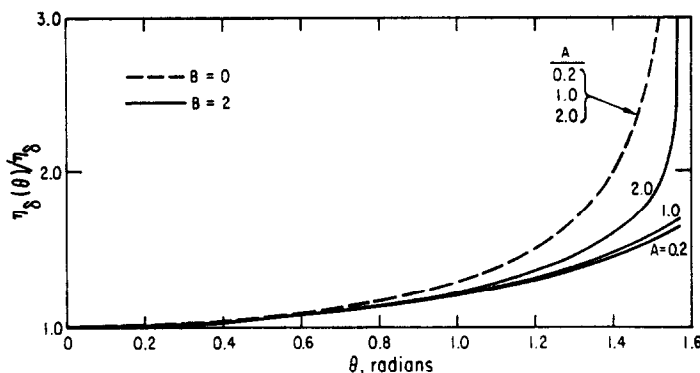

FIG. 3. Variation of local film thickness over the lower surface of a sphere according to equation (43).

a saturated liquid on a flat plate [14]. The liquid drag is not included in equation (43) which is based on a stationary vapor–liquid interface. It may be shown, however, that the angle $[(\pi/2) - \theta]$ in which liquid drag dominates is of the order $4\varepsilon\eta_\delta^{-2}$ and for nearly saturated liquids is insignificant. As mentioned earlier, vapor film behavior under large liquid subcooling is controlled by the heat conducted through the vapor film to the vapor–liquid interface. In this limit the vapor film thickness is obtained by setting the conduction heat flux within the film equal to the solution for heat (or mass) transfer from a sphere in potential flow, as presented by Levich [15] or Sideman [16] and used in expression (43). It has been pointed out in [10] that for highly subcooled liquids almost all the heat arriving at the vapor–liquid interface is convected away in the liquid.

Some care must be exercised in neglecting the effects of vapor film inertia and convection [see Section 2.1, (A1)] as these effects become significant for thick vapor films. Inertia and convection effects are estimated by numerically integrating equations (13) and (14) for the special case of saturated film boiling, $B = 0$. Once again, in this limit the force governing the vapor motion is mainly due to the potential flow pressure distribution so that $f'(\eta_\delta)$ may be taken as zero and the liquid flow field may be ignored. The remaining appropriate boundary conditions are $f(0) = f'(0) = \theta(0) = 0$ and $\theta(\eta_\delta) = 1.0$, $A\theta'(\eta_\delta) = f(\eta_\delta)$. We find that the effects of vapor film inertia and convection on heat-transfer rate is characterized by the correction factor $(1 + 0.9A)^{1/4}$ for the Nusselt number, if the value of $Pr \simeq 1.0^*$. This correction factor is a 'fit' to the numerical results. Since the presence of liquid subcooling only reduces the vapor film thickness, this correction will overestimate the error when B is different from zero. Interestingly enough, for sufficiently large values of A, say $A = 3.0$, neglecting vapor film inertia and convection underestimates Nu by up to 38%, which almost exactly compensates for the error introduced by assuming a uniform vapor film thickness.

From equation (37) we expect the vapor film to be very thin compared with the radius of the sphere if

$$\frac{\delta}{R} = \left(\frac{2}{3}\right)^{1/2}\frac{\beta\eta_\delta}{(Re)^{1/2}} \ll 1.0. \qquad (44)$$

This condition is always satisfied in forced-convection film boiling. The left-hand side of inequality (44) falls in the range $10^{-3}–10^{-2}$. For laminar vapor flow assumption (A3) implies that the vapor film Reynolds number $u\delta/v \simeq \eta_\delta^3 Re^{1/2}/(48\beta)$ be less than ~400. The vapor film Reynolds number is usually of the order of 1.0–150. Except for film boiling of highly subcooled liquids, film Reynolds numbers > 30 are predicted so that wave motion may be superposed on the forward motion of the vapor film. The effective thickness of the thermal boundary layer in the liquid is $R(RePr_l)^{-1/2}$.

Substitution of the typical values $R \simeq 1.0$ cm, $Pr_l \simeq 5.0$, and $Re \simeq 2 \times 10^4$, yields 0.003 cm for the thickness of the thermal layer, a value which is quite small compared to the radius of the sphere [assumption (A6)]. The condition for the neglect of gravity effects is $V_x > (2gR)^{1/2}$ (for example see [1]), or $Re > (8gR^3/v_l^2)^{1/2}$. Putting $R \simeq 1.0$ cm and $v_l \simeq 0.01$ cm^2 s^{-1} gives $Re = 9 \times 10^3$ for the lower bound to the liquid Reynolds number consistent with negligible buoyancy force. All the experimental results presented in Fig. 2 satisfy this criterion. Finally, we come to the assumption of negligible thermal radiation heat transfer [see (A8)]. The maximum Nusselt number for radiative transport is $Nu = 2R\sigma T_w^3/k$, where σ is the Stefan–Boltzman constant. The contribution of radiative heat transfer to the total Nusselt number is found to be less than 10 percent for the conditions reported in [1–4, 7].

5. CONCLUSIONS

In the present paper, the viscous motion generated within a subcooled liquid undergoing forced-convection film boiling in the stagnation region of a sphere (or a cylinder) has been studied. This boiling process may be characterized as follows:

1. For large amounts of subcooling in the liquid, the viscous shear in the liquid becomes vanishingly small. In this thin vapor film limit, heat transfer to the liquid can be evaluated by assuming the existence of a thermal boundary layer superposed on potential flow. The vapor film thickness is strictly controlled by the energy conducted through it to the flowing liquid.

2. For film boiling in nearly saturated liquids, viscous shear effects influence the liquid motion, but exert little influence on the heat transfer. Because of the low density of the vapor compared to that of the liquid, it is the potential flow pressure distribution that drives the vapor motion and not the interfacial shear forces. In addition, almost all the energy transferred from the sphere surface is used to produce vapor and very little energy is transmitted to the viscous liquid.

3. The heat transfer relation (38) derived for the stagnation region is approximately applicable over most of the surface of the sphere not covered by the thick vapor wake. It is found that expression (38), when modified by a numerical correction factor of about 2.0, namely

$$\frac{\beta Nu}{(Re)^{1/2}} = 2.5\left[\frac{1}{24A} + \left(\frac{2}{\pi}\right)^2\left(\frac{B}{A}\right)^4\right]^{1/4}, \qquad (45)$$

provides a reasonable correlation of observed heat-transfer rates for subcooled forced-convection film boiling from spheres or cylinders.

---

* This correction factor is little different than that suggested by Bromley, Leroy and Robbers [1]; viz, $(1 + 0.4A)^{1/2}$.

MICHAEL EPSTEIN and GEORGE M. HAUSER

This work was performed under the auspices of the U.S. Department of Energy.

REFERENCES

1. L. A. Bromley, N. R. LeRoy and J. A. Robbers, Heat transfer in forced convection film boiling, *Ind. Engng Chem.* **45**, 2639 (1953).
2. L. A. Bromley, Heat transfer in stable film boiling, *Chem. Engng Prog.* **46**, 221 (1950).
3. E. I. Motte, Film boiling of flowing subcooled liquids, M.S. Thesis, University of California, Berkeley; University of California Radiation Laboratory Report UCRL-2511 (1954).
4. E. I. Motte and L. A. Bromley, Film boiling of flowing subcooled liquids, *Ind. Engng Chem.* **49**, 1921 (1957).
5. K. Kobayasi, Film boiling heat transfer around a sphere in forced convection, *J. Nucl. Sci.* **2**, 62 (1965).
6. L. C. Witte, Film boiling from a sphere, *Ind. Engng Chem. Fundamentals* **7**, 517 (1968).
7. V. K. Dhir and G. P. Purohit, Subcooled film-boiling heat transfer from spheres, *Nucl. Engng Des.* **47**, 49 (1978).
8. F. J. Walford, Transient heat transfer from a hot nickel sphere moving through water, *Int. J. Heat Mass Transfer* **12**, 1621 (1969).
9. J. W. Stevens and L. C. Witte, Destabilization of vapor film boiling around spheres, *Int. J. Heat Mass Transfer* **16**, 669 (1973).
10. L. C. Witte, L. Baker and D. R. Haworth, Heat transfer from spheres into subcooled liquid sodium during forced convection, *J. Heat Transfer* **90**, 394 (1968).
11. A. G. Guy, *Elements of Physical Metallurgy*. Addison-Wesley, Reading, Mass. (1959).
12. L. C. Witte, J. C. Hesson, M. B. Silverman and R. O. Ivins, Thermal reactor safety studies, in *Chemical Engineering Division Semiannual Report*, Argonne National Laboratory, ANL-7425, p. 152 (1968).
13. I. Shekriladze and Sh. Mestvirishvili, High-rate condensation process theory of vapour flow inside a vertical tube, *Int. J. Heat Mass Transfer* **16**, 715 (1973).
14. R. D. Cess and E. M. Sparrow, Film boiling in a forced-convection boundary-layer flow, *J. Heat Transfer* **83**, 370 (1961).
15. V. G. Levich, *Physicochemical Hydrodynamics*. Prentice-Hall (1962).
16. S. Sideman, The equivalence of the penetration and potential flow theories, *Ind. Engng Chem.* **58**(2), 55 (1966).

EBULLITION EN FILM D'UN LIQUIDE SOUS-REFROIDI
ET EN CONVECTION FORCEE, AU POINT D'ARRET AMONT D'UNE
SPHERE OU D'UN CYLINDRE

Résumé — On analyse l'ébullition en film au point d'arrêt dans les liquides sous-refroidis. Le rôle de la viscosité du liquide dans l'ébullition en film est déterminé en admettant l'existence d'une couche limite hydrodynamique superposée à l'écoulement potentiel et en utilisant une technique de perturbation. La couche visqueuse due à la contrainte tangentielle à l'interface vapeur–liquide perturbe le champ des vitesses légèrement, seulement aux forts sous-refroidissements du liquide. Alors que la solution à viscosité nulle ne peut être utilisée pour décrire le mouvement du liquide quand la température du liquide est proche de sa température de saturation, la vapeur se meut seulement sous l'influence du champ de pression de l'écoulement potentiel, éliminant le couplage entre la couche limite du liquide et le film de vapeur sans erreur appréciable dans le problème du transfert thermique. Une formule d'interpolation rationnelle entre ces deux cas limite conduit à une expression simple pour le transfert thermique accompagnant l'ébullition, en tenant compte des effets principaux de la surchauffe de la paroi et du sous-refroidissement du liquide. On démontre l'applicabilité de cette formule à l'ébullition en film à partir d'une sphère ou d'un cylindre.

UNTERKÜHLTES FILMSIEDEN BEI ERZWUNGENER KONVEKTION IM
VORDEREN STAUGEBIET EINER KUGEL ODER EINES ZYLINDERS

Zusammenfassung – Filmsieden bei erzwungener Konvektion in der Staustiömung unterkühlter Flüssigkeiten wird untersucht. Mittels eines Störungsansatzes wird unter Voraussetzung der Existenz einer der Potentialströmung überlagerten hydrodynamischen Grenzschicht der Einfluß der Flüssigkeitsviskosität auf das Filmsieden bestimmt. Es wird gezeigt, daß die durch die Schubspannung an der Dampf-Flüssigkeits-Grenzfläche bedingte Reibungs-Grenzschicht bei großer Flüssigkeits-Unterkühlung das Geschwindigkeitsfeld nur geringfügig stört. Während zur Beschreibung der Flüssigkeitsbewegung die reibungsfreie Lösung nicht verwendet werden kann, wenn die Flüssigkeit annähernd Sättigungstemperatur hat, ist die Dampfbewegung nur von der Potentialströmungs-Druckverteilung abhängig. Dadurch ergibt sich ohne wesentliche Fehler bezüglich der Wärmeübertragung eine Entkopplung zwischen der Flüssigkeitsgrenzschicht und dem Dampffilm. Eine zweckmäßige Interpolationsformel zwischen diesen beiden Grenzfällen führt auf einen einfachen Ausdruck für den Wärmeübergang beim Filmsieden in Abhängigkeit von den Haupteinflußgrößen der Wandüberhitzung und der Flüssigkeitsunterkühlung. Die Anwendbarkeit dieser Beziehung auf unterkühltes Filmsieden an einer Kugel oder einem Zylinder wird gezeigt.

ПЛЁНОЧНОЕ КИПЕНИЕ С НЕДОГРЕВОМ В ПЕРЕДНЕЙ КРИТИЧЕСКОЙ ЗОНЕ ДЛЯ ШАРА ИЛИ ЦИЛИНДРА ПРИ ВЫНУЖДЕННОЙ КОНВЕКЦИИ

Аннотация — Проведен анализ плёночного кипения недогретой жидкости в критической зоне при вынужденной конвекции. Влияние вязкости жидкости на плёночное кипение определяется с помощью предположения о наличии гидродинамического пограничного слоя, наложенного на потенциальное течение, и использования метода возмущений. Показано, что в силу наличия сдвигового напряжения на поверхности раздела пар-жидкость пограничный слой только незначительно возмущает поле скорости при больших значениях недогрева жидкости. Хотя для описания течения жидкости в случае, когда её температура приближается к температуре насыщения, нельзя использовать решение для невязкого случая, найдено, что пар движется только под влиянием распределения давления в потенциальном потоке, что позволяет, без внесения существенных погрешностей в задачу о теплопереносе, рассматривать отдельно пограничный слой жидкости и паровую плёнку. Использование рациональной интерполяционной формулы для описания этих двух предельных случаев позволяет получить простое выражение для теплопереноса при плёночном кипении, которое учитывает такие основные эффекты, как перегрев стенки и недогрев жидкости. Показана применимость формулы к случаю плёночного кипения с недогревом на шаре или цилиндре.